

R^n Extension of Starobinsky Model in Old Minimal Supergravity

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ABSTRACT

We provide a succinct way to construct the supersymmetric completion of R^n ($n \geq 3$) in components using superconformal formulation of old minimal supergravity. As a consequence, we obtain the polynomial $f(R)$ supergravity extending the supersymmetric Starobinsky model to any higher power of R .

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1 Introduction

If the perturbations during inflation [1] are originated by the same field driving inflation, then the recent Planck data on the cosmic microwave background (CMB) radiation anisotropies have severely constrained the models of single-field inflation [2]. Successful models have to predict a significant red tilt in the power spectrum of the scalar curvature perturbation, measured by the spectral index $n_s = 0.960 \pm 0.007$, and a low enough amount of tensor perturbations quantified by the current bound on the tensor-to-scalar ratio, $r < 0.08$. One of the models which better passes these constraints is the R^2 Starobinsky model [3, 4]. It is described by the action ($M_{\text{pl}} = 1$)

$$S = \frac{1}{2} \int d^4x \sqrt{-g} \left(R + R^2/(6M^2) \right), \quad (1.1)$$

where the positivity of the coefficient in front of R^2 term is required to avoid instabilities. The inflaton with mass M is given by the spin-0 part of the metric and thus has a clear geometric origin. CMB data determines the mass parameter M to be $M = (3.0 \times 10^{-6}) \frac{50}{N_e}$, with N_e being the e-folding number.

It is natural to consider embedding Starobinsky model in supergravity since supersymmetry is considered to be the leading proposal of new physics beyond the Standard Model. Old and recent implementation of the Starobinsky model can be found in old minimal supergravity in [5, 6, 7, 8] and in new minimal supergravity in [9, 10, 11]. These theories have the common feature of adding to pure supergravity four bosonic and four fermionic degrees of freedom. At the linearized level the $4_b + 4_f$ degrees of freedom comprise two massive chiral multiplets in the old minimal formulation [12] and one massive vector multiplet in

the new minimal formulation. Another class of supersymmetric embedding of Starobinsky with a single chiral superfield coupled to supergravity was considered in [13]. Elimination of the auxiliary field could produce R^2 or higher powers. However, it has been shown that [14, 15] this class of model is insufficient for realization of the Starobinsky inflation .

In this paper, we investigate the possibility of adding higher order corrections to the supersymmetric extension of Starobinsky model in the old minimal formulation. For the absence of ghosts, we consider these higher order corrections to be the supersymmetric completion of R^n . Utilizing 4D $\mathcal{N} = 1$ superconformal tensor calculus, we obtain the supersymmetric $R^n (n \geq 3)$ action in its component form explicitly. Summing over all these R^n actions leads to an extension of Starobinsky model to any order in terms of the power of R . Realization of slow roll inflation probably would restrict the higher order parameters to be sufficiently small. However, it is still interesting to explore the physical consequences implied by the higher order corrections which are accumulated during the long history of the universe.

2 Settings

In this section, we introduce the necessary ingredients needed in the construction of the supersymmetric R^n action. The 4D $\mathcal{N} = 1$ superconformal tensor calculus is based on the superconformal algebra $SU(2, 2|1)$ [16]. The gauge fields associated with general coordinate transformations (e_μ^a), dilatations (b_μ), chiral $U(1)$ symmetry (A_μ) and Q supersymmetry (ψ_μ^α) are independent fields comprising the $12_b + 12_f$ 4D $\mathcal{N} = 1$ superconformal Weyl multiplet. The remaining gauge fields associated with the Lorentz transformation ω_μ^{ab} , special conformal transformation (f_μ^a) and S supersymmetry transformation (ϕ_μ^α) are dependent fields. They are composite objects, which depend on the independent fields of the multiplet. We refer to [17] for an extended summary of the superconformal transformations of the Weyl multiplet fields, the expressions for the curvatures and other useful details.

In the superconformal method for $\mathcal{N} = 1$ supergravity, a compensating chiral multiplet is used to fix the extra superconformal symmetries, giving rise to the standard old minimal formulation of Poincaré supergravity. Chiral multiplets also provide matter coupling to conformal supergravity. Generically, a chiral multiplet consists of two complex scalars Z and \mathcal{F} , and the left-chiral projection of a Majorana spinor $P_L\chi$. Totally, these fields carry $4_b + 4_f$ off shell degrees of freedom. The Q - and S - supersymmetry transformation rules of

chiral multiplet are given in [17]

$$\begin{aligned}
\delta Z &= \bar{\epsilon} P_L \chi, \\
\delta P_L \chi &= \frac{1}{2} P_L (\not{D} Z + \mathcal{F}) \epsilon + \omega_Z Z P_L \eta, \\
\delta \mathcal{F} &= \bar{\epsilon} \not{D} P_L \chi - 2(\omega_Z - 1) \bar{\eta} P_L \chi,
\end{aligned} \tag{2.1}$$

where ω_Z is the conformal weight of Z . The closure of supersymmetry algebra requires that the chiral $U(1)$ charge of Z equals its conformal weight. The supercovariant derivatives appearing in the transformation rules takes the form [17]

$$\begin{aligned}
\mathcal{D}_\mu Z &= (\partial_\mu - \omega_Z b_\mu - i\omega_Z A_\mu) Z - \bar{\psi}_\mu P_L \chi, \\
\mathcal{D}_\mu P_L \chi &= P_L \left[\left(\partial_\mu - (\omega_Z + \frac{1}{2}) b_\mu + \frac{1}{4} \omega_\mu^{ab} \gamma_{ab} \right. \right. \\
&\quad \left. \left. - (\omega_Z - \frac{3}{2}) i A_\mu \right) \chi - \frac{1}{2} (\not{D} Z + \mathcal{F}) \psi_\mu \right. \\
&\quad \left. - \omega_Z Z \phi_\mu \right].
\end{aligned} \tag{2.2}$$

Since a chiral multiplet is identified by its first component, we therefore use its first component to stand for the whole multiplet.

3 Construction of the Supersymmetric R^n Action

To construct the supersymmetric completion of R^n , we first introduce the component form of fusion rule satisfied by the chiral multiplets. Based on the fusion rule, one can construct a composite chiral multiplet by using two known chiral multiplets. Given the fact that chiral multiplets in 4D $\mathcal{N} = 1$ conformal supergravity can have arbitrary Weyl weights, fusion rule is able to be utilized flexibly which facilitates the supersymmetrization of R^n tremendously. Without referring to the cumbersome superspace formulation, we obtain the bosonic part of supersymmetric R^n action straightforwardly. In particular, we attain the supersymmetric R^3 action which is beyond the construction adopted in [11, 18].

In order to achieve the R^n supergravity, we utilize four different chiral multiplets, which are given in Table 1 along with the Weyl weights and chiral $U(1)$ charges of their lowest components. From (2.2), we can see that their supersymmetry transformation rules are determined by the Weyl weights and chiral $U(1)$ charges of the lowest components. The highest weight component of the auxiliary multiplet has Weyl weight 4, therefore its superconformal completion can be used as the Lagrangian density. Its full expression is given in [17]

$$e^{-1} \mathcal{L}_A = \text{Re} \left(N + \sqrt{2} \bar{\psi}_\mu \gamma^\mu P_L \Omega + \frac{1}{2} A \bar{\psi}_\mu \gamma^{\mu\nu} P_R \psi_\nu \right). \tag{3.1}$$

TABLE 1

List of Chiral multiplets used in the construction of the R^n extension of the Starobinsky Model.

Name	Weyl weight	Chiral weight	Components
Neutral	0	0	$(\sigma, P_L\psi, F)$
Compensating	1	1	$(\phi, P_L\lambda, S)$
Curvature	2	2	$(\tilde{S}, \not{D}P_L\lambda, \square^C\bar{\phi})$
Auxiliary	3	3	$(A, P_L\Omega, N)$

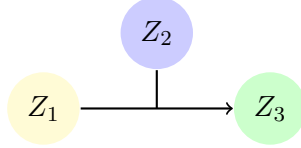
3.1 Fusion Rule and the Map Between a Chiral and the Weyl Multiplet

In superspace formulation, the multiplication of two chiral superfield gives rise to a chiral superfield. In terms of components, this means [17]

$$\begin{aligned}
Z_3 &= Z_1 Z_2, \\
P_L\chi_3 &= Z_1 P_L\chi_2 + Z_2 P_L\chi_1, \\
\mathcal{F}_3 &= Z_1 \mathcal{F}_2 + Z_2 \mathcal{F}_1 - 2\bar{\chi}_1 P_L\chi_2,
\end{aligned} \tag{3.2}$$

where the conformal weight of Z_3 equals the sum of the conformal weights of Z_1 and Z_2 . This fusion rule is visualized in Fig(1).

FIGURE 1
Fusion of two chiral multiplets



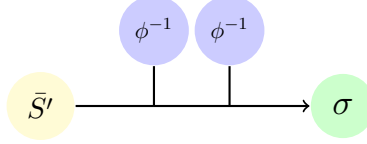
It can be deduced from (3.2) that when multiple chiral multiplets are composed into a new chiral multiplet, the order of the fusion will not affect the result. In other words, if $Z_{i_1 \dots i_n}$ results from the successive fusions of $Z_{i_1}, Z_{i_2} \dots Z_{i_n}$, then $Z_{i_1 \dots i_n}$ is totally symmetric with respect to indices $i_1 \dots i_n$.

The fusion rule (3.2) can be inverted and lead to another fusion rule for chiral multiplet

$$\begin{aligned}
Z_1 &= Z_3 Z_2^{-1}, \\
P_L\chi_1 &= Z_2^{-1} P_L\chi_3 - Z_3 Z_2^{-2} P_L\chi_2, \\
\mathcal{F}_1 &= Z_2^{-1} \mathcal{F}_3 - Z_3 Z_2^{-2} \mathcal{F}_2 + 2Z_2^{-2} \bar{\chi}_2 P_L\chi_3 \\
&\quad - 2Z_3 Z_2^{-3} \bar{\chi}_2 P_L\chi_2.
\end{aligned} \tag{3.3}$$

Based on the second fusion rule (3.3), a neutral multiplet σ can be formed by using 2 compensating multiplet ϕ and the curvature multiplet $(\bar{S}', \not{D}P_L\lambda', \square^C\bar{\phi}')$. The formation of the composite neutral multiplet is illustrated in Fig(2). Here we use “prime” to distinguish the weight-1 anti-chiral multiplet $\bar{\phi}'$ with the complex conjugate of the compensating multiplet ϕ . In general, they are independent of each other.

FIGURE 2
The formaton of neutral multiplet



As a consequence, the bosonic composite expressions are

$$\sigma = \phi^{-2}\bar{S}', \quad F = \phi^{-2}\square^C\bar{\phi}' - 2\phi^{-3}\bar{S}'S. \quad (3.4)$$

For briefness and later use, from now on, we only present the purely bosonic terms in the expressions of the composite chiral multiplet. If the anti-chiral multiplet $\bar{\phi}'$ happens to be the complex conjugate of the compensating multiplet ϕ , we can derive a map between the neutral multiplet and the Weyl multiplet after eliminating the extra superconformal symmetry by gauge fixing.

We choose the following gauge conditions to fix dilatation symmetry, local chiral $U(1)$ symmetry, S supersymmetry and special conformal symmetry,

$$\phi = \sqrt{3}, \quad \lambda = 0, \quad b_\mu = 0, \quad (3.5)$$

which also leads to the map from the neutral multiplet to the supergravity multiplet

$$\begin{aligned} \sigma &= -\frac{1}{\sqrt{3}}\bar{S}, \\ F &= \frac{1}{6}R + A_\mu A^\mu - i(\nabla_\mu A^\mu) + \frac{2}{3}\bar{S}S, \end{aligned} \quad (3.6)$$

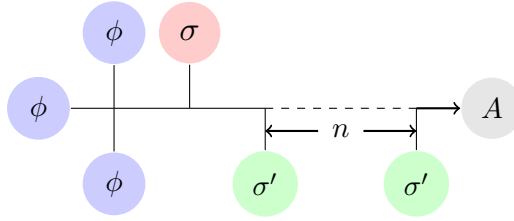
where R enters through $\square^C\bar{\phi}$ and the resulting chiral multiplet is only manifestly Poincaré invariant. It should be noted that F contains the Ricci scalar, and S now becomes the auxiliary scalar in the old minimal Poincaré supergravity, which is stated by its transformation rule

$$\delta S = \frac{1}{\sqrt{2}}\bar{\epsilon}P_R\gamma^{\mu\nu}\hat{\psi}_{\mu\nu}. \quad (3.7)$$

3.2 Supersymmetric Completion of R^n Action

With previous preparations, we now proceed with the supersymmetrization of R^n action. We will utilize the fact that the neutral chiral multiplet σ carries 0 Weyl weight and chiral charge. Therefore, according to the fusion rule (3.2), it can be combined with a second chiral multiplet to form a third chiral multiplet which shares the same properties with the second chiral multiplet. This fusion can be duplicated for arbitrary times. At the end of each fusion, we arrive at a different composite multiplet, however, the conformal weight and chiral $U(1)$ charge remain unchanged. Considering that the superconformal action (3.1) is associated with the auxiliary multiplet, we need to construct the auxiliary multiplet in terms of the neutral multiplet and the compensating multiplet. The former brings the R dependence through F , while the latter balances the conformal weight and chiral charge. By a process of trial and error, we find that to obtain the supersymmetrization of R^n , two independent neutral multiplets are needed. Fusing 3 compensating multiplets ϕ , $n - 1$ of the first neutral multiplets denoted by σ' and 1 of the second neutral multiplet represented by σ leads to the composite expression for the auxiliary multiplet

FIGURE 3
Fusion rule of an auxiliary multiplet for the construction of R^n action



$$A = \phi^3 \sigma (\sigma')^{n-1}, \quad N = (\sigma')^{n-1} (3\phi^2 \sigma S + \phi^3 F) + (n-1) (\sigma')^{n-2} \sigma \phi^3 F'. \quad (3.8)$$

Then, using the auxiliary chiral multiplet action (3.1), we obtain the corresponding Lagrangian as

$$e^{-1} \mathcal{L} = \text{Re} \left[(\sigma')^{n-1} (3\phi^2 \sigma S + \phi^3 F) + (n-1) (\sigma')^{n-2} \sigma \phi^3 F' \right]. \quad (3.9)$$

In the above expressions, F can be replaced according to (3.6) which includes the Ricci scalar. Thus, if the neutral multiplet σ' can be realized in terms of the neutral multiplet σ in such a way that σ' depends on F linearly, we are capable of deriving the supersymmetric R^n action. This can be achieved by two steps. We first combine the neutral multiplet σ

with the compensating multiplet ϕ to form a new weight-1 chiral multiplet ϕ'

$$\phi' = \sigma\phi, \quad S' = \sigma S + \phi F. \quad (3.10)$$

We then substitute (3.10) into (3.4). Finally, the neutral multiplet σ' is given as the composite constructed from the neutral multiplet σ and the compensating multiplet ϕ

$$\begin{aligned} \sigma' &= \phi^{-2}(\bar{\sigma}\bar{S} + \bar{\phi}\bar{F}), \\ F' &= \phi^{-2}\square^C(\bar{\phi}\bar{\sigma}) - 2\phi^{-3}\bar{\sigma}S\bar{S} - 2\phi^{-3}\bar{\phi}S\bar{F}, \end{aligned} \quad (3.11)$$

where σ' depends on F linearly as required. Plugging (3.11) into (3.9), fixing the redundant superconformal symmetry and replacing σ , F according to (3.6) completes the supersymmetrization of R^n action whose bosonic part is given by

$$\begin{aligned} e^{-1}\mathcal{L}_{R^n} &= \text{Re} \left[\left(R + 6A^\mu A_\mu + 6i\nabla_\mu A^\mu + 2S\bar{S} \right)^{n-2} \times \left(\frac{1}{12}R^2 + RA^\mu A_\mu + \frac{n-1}{6}RS\bar{S} \right. \right. \\ &\quad \left. \left. + 3(A^\mu A_\mu)^2 + \frac{2n-3}{3}(S\bar{S})^2 + (n-1)A^\mu A_\mu S\bar{S} + 3(\nabla_\mu A^\mu)^2 + (n-1)\bar{S}\square S \right. \right. \\ &\quad \left. \left. + (3n-5)iS\bar{S}(\nabla_\mu A^\mu) + (2n-2)iA_\mu \bar{S}\partial^\mu S \right) \right]. \end{aligned} \quad (3.12)$$

For $n = 1$, (3.12) gives rises the old minimal Poincaré supergravity

$$e^{-1}\mathcal{L}_R = R + 6A^\mu A_\mu - 2S\bar{S}. \quad (3.13)$$

For $n = 2$, (3.12) reproduces the supersymmetric R^2 action in old minimal supergravity [5]

$$e^{-1}\mathcal{L}_{R^2} = R^2 + 12RA^\mu A_\mu + 2R\bar{S}S + 36(\nabla_\mu A^\mu)^2 - 12D_\mu \bar{S}D^\mu S + 4(S\bar{S} + 3A^\mu A_\mu)^2, \quad (3.14)$$

where we have defined $D_\mu S \equiv \partial_\mu S + iA_\mu S$. To arrive at (3.13) and (3.14), an overall scaling of the Lagrangian has been performed.

3.3 Cosmological Constant

The supersymmetric cosmological constant term can be derived by constructing an auxiliary multiplet using 3 compensating multiplets. This amounts to set $\sigma = \sigma' = 1$ and $F = F' = 0$ in (3.9). Upon eliminating the extra conformal symmetries, the supersymmetric cosmological constant term takes the simple form

$$\mathcal{L}_\Lambda = \text{Re}(S). \quad (3.15)$$

The vacuum expectation value of S behaves as a cosmological constant parameter appearing in the Lagrangian. However, in a higher curvature gravity theory, the higher derivative interactions can generate cosmological constant. Thus the effective cosmological constant should be read off from the value of the curvature tensor solved from the equation of motion.

4 Discussion

We have constructed the supersymmetric completion of R^n action in the old minimal formulation of 4D $\mathcal{N} = 1$ supergravity, extending the Starobinsky model to arbitrary power of R in a supersymmetric pattern. The action is given by

$$e^{-1}\mathcal{L} = R + e^{-1}\mathcal{L}_{R^2}/(6M^2) + \sum_{n \geq 3} \xi_n e^{-1}\mathcal{L}_{R^n}, \quad (4.1)$$

where the supersymmetric R^n action can be found in (3.12). The construction used 3 compensating multiplets ϕ , $n-1$ of neutral multiplet σ' and 1 neutral multiplet σ . Instead, if considering $p-1$ number of σ' and q number of σ , one can obtain the dilatonic R^n action as

$$e^{-1}\mathcal{L}_{R^p S^{q-1}} = \text{Re} \left[(\sigma')^{p-1} (3\phi^2 \sigma^q S + q\phi^3 \sigma^{q-1} F) + (p-1)(\sigma')^{p-2} \sigma^q \phi^3 F' \right], \quad (4.2)$$

with σ', F' being replaced by (3.11) and σ, F being further replaced by (3.6). It then can be seen that in (4.2) the term with highest power of R is given by $R^p(S^{q-1} + \bar{S}^{q-1})$. Adding these dilatonic R^n invariants to the action (4.1) may bring novel feature to the inflation model which is worth a future study.

The R^n extended Starobinsky model possesses a dual description in terms of the standard supergravity coupled to two chiral multiplets model. In fact, our construction is closely related to the dual model of the supersymmetric R^n extension of Starobinsky model. The dual model corresponding to (4.1) can be derived as follows. After plugging (3.11) into (3.9) and fixing the redundant superconformal symmetries, we do not replace σ, F according to (3.6) undoing the step leading to the R^n action. Then Lagrangian multipliers should be added to the Lagrangian whose equations of motion imply the map from chiral multiplet σ to the supergravity multiplet (3.6). Integrating out the auxiliary fields of the supergravity multiplet and performing a Weyl scaling on the metric eventually gives rises to the standard supergravity coupled to two chiral multiplets.

In the R^n extended Starobinsky model, the scalars participating inflation are the spin-0 part of the metric, the longitudinal mode of A_μ and the complex scalar S . The Lagrangian (3.12) seems to suggest that rather complicated interactions are involved among these scalars. In terms of the dual model, this would imply very sophisticated form of the scalar potential. The polynomial $f(R)$ extended model (4.1) contains infinite number of parameters, thus its dual model is capable of describing nearly arbitrary inflation potentials. The stability of the extra scalar modes and universality class of inflation in the

supersymmetric R^n extended model should be interesting and deserve further efforts to uncover.

Acknowledgements

The work of Y.P. is supported in part by DOE grant DE-SC0010813.

References

- [1] D. H. Lyth and A. Riotto, *Particle physics models of inflation and the cosmological density perturbation*, Phys. Rept. **314**, 1 (1999), hep-ph/9807278.
- [2] P. A. R. Ade *et al.* [Planck Collaboration], *Planck 2013 results. XXII. Constraints on inflation*, arXiv:1303.5082 [astro-ph.CO].
- [3] A. A. Starobinsky, *A New Type of Isotropic Cosmological Models Without Singularity*, Phys. Lett. B **91**, 99 (1980).
- [4] V. F. Mukhanov and G. V. Chibisov, *Quantum Fluctuation and Nonsingular Universe. (In Russian)*, JETP Lett. **33**, 532 (1981) [Pisma Zh. Eksp. Teor. Fiz. **33**, 549 (1981)].
- [5] S. Cecotti, *Higher Derivative Supergravity Is Equivalent To Standard Supergravity Coupled To Matter. 1.*, Phys. Lett. B **190**, 86 (1987).
- [6] J. Ellis, D. V. Nanopoulos and K. A. Olive, *No-Scale Supergravity Realization of the Starobinsky Model of Inflation*, Phys. Rev. Lett. **111**, 111301 (2013), arXiv:1305.1247 [hep-th].
- [7] R. Kallosh and A. Linde, *Superconformal generalizations of the Starobinsky model*, JCAP **1306**, 028 (2013), arXiv:1306.3214 [hep-th].
- [8] W. Buchmuller, V. Domcke and K. Kamada, *The Starobinsky Model from Superconformal D-Term Inflation*, Phys. Lett. B **726**, 467 (2013), arXiv:1306.3471 [hep-th].
- [9] S. Cecotti, S. Ferrara, M. Porrati and S. Sabharwal, *New Minimal Higher Derivative Supergravity Coupled To Matter*, Nucl. Phys. B **306**, 160 (1988).
- [10] M. de Roo, A. Wiedemann and E. Zijlstra, *The Construction of R^2 Actions in $D = 4$, $N = 1$ Supergravity*, Class. Quant. Grav. **7**, 1181 (1990).

- [11] S. Ferrara, R. Kallosh, A. Linde and M. Porrati, *Higher Order Corrections in Minimal Supergravity Models of Inflation*, JCAP **1311**, 046 (2013), arXiv:1309.1085 [hep-th].
- [12] S. Ferrara, M. T. Grisaru and P. van Nieuwenhuizen, *Poincare and Conformal Supergravity Models With Closed Algebras*, Nucl. Phys. B **138**, 430 (1978).
- [13] S. V. Ketov and A. A. Starobinsky, *Embedding $(R + R^2)$ -Inflation into Supergravity*, Phys. Rev. D **83**, 063512 (2011), arXiv:1011.0240 [hep-th].
- [14] S. Ferrara, R. Kallosh and A. Van Proeyen, *On the Supersymmetric Completion of $R + R^2$ Gravity and Cosmology*, JHEP **1311**, 134 (2013), arXiv:1309.4052 [hep-th].
- [15] S. V. Ketov and T. Terada, *Old-minimal supergravity models of inflation*, JHEP **1312**, 040 (2013), arXiv:1309.7494 [hep-th].
- [16] S. Ferrara and P. van Nieuwenhuizen, *The Auxiliary Fields of Supergravity*, Phys. Lett. B **74**, 333 (1978).
- [17] D.Z. Freedman and A. Van Proeyen, *Supergravity*, Cambridge University Press 2012.
- [18] F. Farakos, A. Kehagias and A. Riotto, *On the Starobinsky Model of Inflation from Supergravity*, Nucl. Phys. B **876**, 187 (2013), arXiv:1307.1137.